4.2 Graphs of Motion

Consider the phrase "a picture is worth a thousand words." A graph is a special kind of picture that can quickly give meaning to a lot of data (numbers). You can easily spot relationships on a graph. It is much more difficult to see these same relationships by looking at columns of numbers. Compare the table of numbers to the graph in Figure 4.4 and see if you agree!

VOCABULARY

constant speed - speed that stays the same.

The position vs. time graph

Recording data Imagine you are helping a runner who is training for a track meet. She wants to know if she is running at a **constant speed**. Constant speed means the speed stays the same. You mark the track every 50 meters. Then you measure her time at each mark as she runs. The data for your experiment is shown in Figure 4.4. This is position vs. time data because it tells you the runner's position at different points in time. She is at 50 meters after 10 seconds, 100 meters after 20 seconds, and so on.

Graphing data To graph the data, you put position on the vertical (y) axis and time on the horizontal (x) axis. Each row of the data table makes one point on the graph. Notice the graph goes over to the right 10 seconds and up 50 meters between each point. This makes the points fall exactly in a straight line. The straight line tells you the runner moves the same distance during each equal time period. *An object moving at a constant speed always creates a straight line on a position vs. time graph.*

Calculating speed The data shows that the runner took 10 seconds to run each 50-meter segment. Because the time and distance was the same for each segment, you know her speed was the same for each segment. You can use the formula $v = d/t$ to calculate the speed. Dividing 50 meters by 10 seconds tells you her constant speed was 5 meters per second.

Position and Time Data for a Runner

Time (s)	Position (m)
0	0
10	50
20	100
30	150

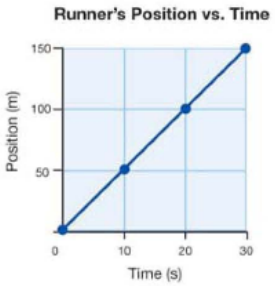


Figure 4.4: *A data table and a position vs. time graph for a runner.*

Graphs show relationships between variables

Relationships between variables

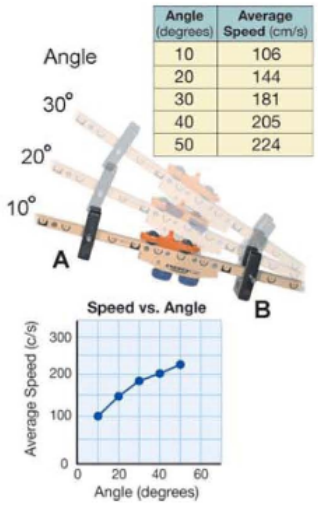
Think about rolling a toy car down a ramp. You theorize that steeper angles on the ramp will make the car go faster. How do you find out if your theory is correct? You need to know the relationship between the variables *angle* and *speed*.

Patterns on a graph show relationships

Recall that in a graph, the dependent variable is usually plotted on the vertical (or *y*) axis and the independent variable is usually on the horizontal (or *x*) axis. Each axis is marked with the range of values the variable has. In Figure 4.5, the *x*-axis (angle) has values between 0 and 60 degrees. The *y*-axis (average speed) has values between 0 and 300 cm/s. You can tell there is a relationship because all the points on the graph follow the same curve that slopes up and to the right. The graph tells you instantly that the average speed increases as the ramp gets steeper.

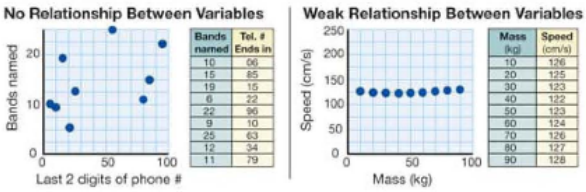
Recognizing a relationship from a graph

Recall that the relationship between variables may be strong, weak, or there may be no relationship at all. In a strong relationship, large changes in one variable make similarly large changes in the other variable, like in Figure 4.5. In a weak relationship, large changes in one variable cause only small changes in the other. The graph on the right (below) shows a weak relationship. When there is no relationship, the graph looks like scattered dots (below left). The dots do not make an obvious pattern (a line or curve).



Angle (degrees)	Average Speed (cm/s)
10	106
20	144
30	181
40	205
50	224

Figure 4.5: *This graph shows that the average speed between A and B increases as the angle of the track increases.*



Bands named	Tel. # Ends in
10	66
15	85
19	15
6	22
22	96
9	10
25	63
12	34
11	79

Mass (kg)	Speed (cm/s)
10	126
20	125
30	123
40	122
50	123
60	124
70	126
80	127
90	128

Slope

Comparing speeds You can use position vs. time graphs to quickly compare speeds. Figure 4.6 shows a position vs. time graph for two people running along a jogging path. Both runners start at the beginning of the path (the origin) at the same time. Runner A (blue) takes 100 seconds to run 600 meters. Runner B (red) takes 150 seconds to go the same distance. Runner A's speed is 6 m/s (600 ÷ 100) and runner B's speed is 4 m/s (600 ÷ 150). Notice that the line for runner A is *steeper* than the line for runner B. A steeper line on a position vs. time graph means a faster speed.

A steeper line on a position vs. time graph means a faster speed.

Calculating slope The "steepness" of a line is called its slope. The **slope** is the ratio of the *rise* (vertical change) divided by the *run* (horizontal change). The diagram below shows how to calculate the slope of a line. Visualize a triangle with the slope as the hypotenuse. The rise is the height of the triangle. The run is the length along the base. Here, the *x*-axis is time and the *y*-axis is position. The slope of the graph is therefore the distance traveled divided by the time it takes, or the speed. The units are the units for the rise (meters) divided by the units for the run (seconds), meters per second, or m/s.

VOCABULARY

slope - the ratio of the rise (vertical change) to the run (horizontal change) of a line on a graph.

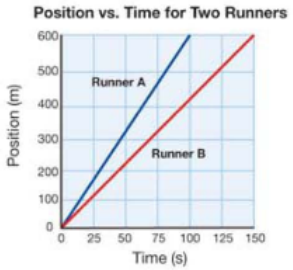
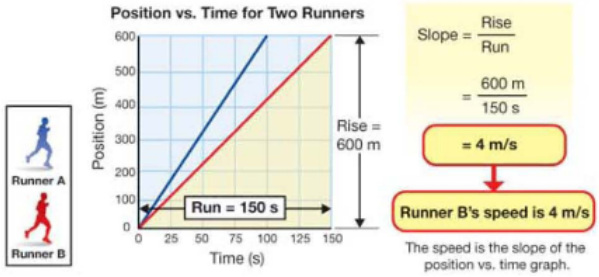


Figure 4.6: *A position vs. time graph for two runners.*



Speed vs. time graphs

Constant speed on a speed vs. time graph

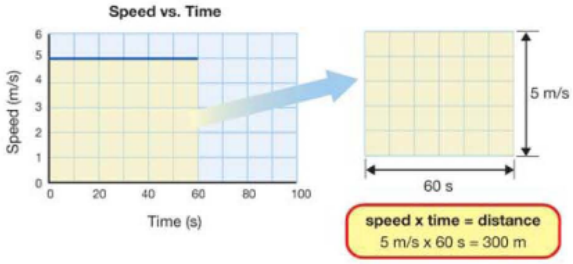
The speed vs. time graph has speed on the *y*-axis and time on the *x*-axis. The bottom graph in Figure 4.7 shows the speed vs. time for the runner. The top graph shows the position vs. time. Can you see the relationship between the two graphs? The blue runner has a speed of 5 m/s. The speed vs. time graph shows a horizontal line at 5 m/s for the entire time. On a speed vs. time graph, constant speed is shown with a straight horizontal line. At any point in time between 0 and 60 seconds the line tells you the speed is 5 m/s.

Another example

The red runner's line on the position vs. time graph has a less steep slope. That means her speed is slower. You can see this immediately on the speed vs. time graph. The red runner shows a line at 4 m/s for the whole time.

Calculating distance

A speed vs. time graph can also be used to find the *distance* the object has traveled. Remember, distance is equal to speed multiplied by time. Suppose we draw a rectangle on the speed vs. time graph between the *x*-axis and the line showing the speed. The area of the rectangle (shown below) is equal to its length times its height. On the graph, the length is equal to the time and the height is equal to the speed. Therefore, the area of the graph is the speed multiplied by the time. This is the distance the runner traveled.



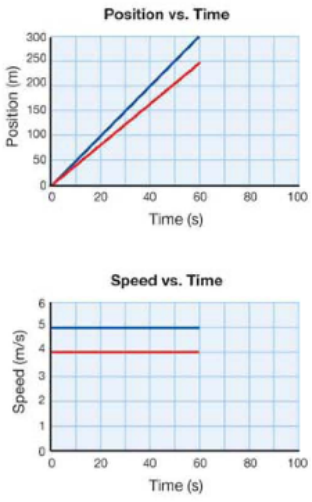
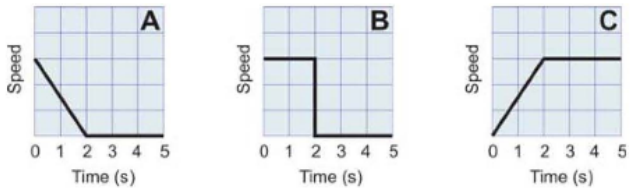


Figure 4.7: *The position vs. time graph (top) shows the exact same motion as the speed vs. time graph (bottom).*

Section 4.2 Review

1. On a graph of position vs. time, what do the *x*-values represent? What do the *y*-values represent?
2. Explain why time is an independent variable and position is a dependent variable in a position vs. time graph.
3. What does the slope of the line on a position vs. time graph tell you about an object's speed?
4. The graph in Figure 4.8 shows the position and time for two runners in a race. Who has the faster speed, Robin or Joel? Explain how to answer this question without doing calculations.
5. Calculate the speed of each runner from the graph in Figure 4.8.
6. The runners in Figure 4.8 are racing. Predict which runner will get to the finish line of the race first.
7. Maria walks at a constant speed of 2 m/s for 8 seconds.
   a. Draw a speed vs. time graph for Maria's motion.
   b. How far does she walk?
8. Which of the three graphs below corresponds to the position vs. time graph in Figure 4.9?



9. Between which times is the speed zero for the motion shown on the position vs. time graph in Figure 4.9?

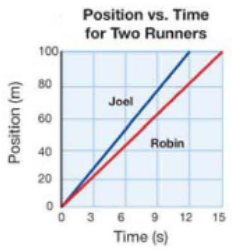


Figure 4.8: *Questions 4, 5, and 6.*

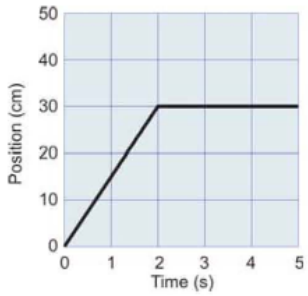


Figure 4.9: *Questions 8 and 9.*